

# A survey of the ESR model for an objective reinterpretation of Quantum Mechanics

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## Abstract

Most scholars concerned with the foundations of quantum mechanics (QM) think that *contextuality* and *nonlocality* (hence *nonobjectivity* of physical properties) are unavoidable features of QM which follow from the mathematical apparatus of QM. Moreover these features are usually considered as basic in quantum information processing. Nevertheless they raise still unsolved problems, as the *objectification problem* in the quantum theory of measurement. The *extended semantic realism (ESR) model* offers a possible way out from these difficulties by embedding the mathematical formalism of QM into a broader mathematical formalism and reinterpreting quantum probabilities as conditional on detection rather than absolute. The embedding allows to recover the formal apparatus of QM within the ESR model, and the reinterpretation of QM allows to construct a noncontextual hidden variables theory which justifies the assumptions introduced in the ESR model and proves its objectivity. According to the ESR model both linear and nonlinear time evolution occur, depending on the physical environment, as in QM. In addition, the ESR model, though objective, implies modified Bell's inequalities that do not conflict with QM, supplies different mathematical representations of proper and improper mixtures, provides a general framework in which the local interpretations of the GHZ experiment obtained by other authors are recovered and explained, and supports an interpretation of quantum logic which avoids the introduction of the problematic notion of quantum truth.

## 1 Introduction.

Since its birth quantum mechanics (QM) proved to be a theory of outstanding empirical success, but also a source of problems and paradoxes. These mainly follow from the proposed interpretations of the theory, which multiplied in time

and are still debated. According to Busch *et al.* [1] these interpretation can be divided in two classes.

(i) *Statistical interpretations*: QM refers to frequencies of measurements outcomes only. No reference to microscopic objects should enter its language.

(ii) *Ontic, or realistic interpretations*: QM deals with items of physical systems, or *individual objects*, and their properties.

The statistical interpretations avoid many problems but can be criticized from several viewpoints. They imply indeed an instrumentalist view and lack explanatory power. Moreover, nowadays experimental physicists often claim that they can deal with individual objects, not only with statistical ensembles.

On the other side, the realistic interpretations can be reformulated avoiding ontological commitments if “individual object” is considered as a term of the theoretical language of QM, interpreted (via observational language) as a click in a preparing device. But in these interpretations, however reformulated, a crucial problem occurs, at least if one does not want to go back to a merely statistical interpretation of QM (now in terms of ensembles of individual objects rather than measurements outcomes): the *nonobjectivity* of physical properties in QM, following from “no-go” theorems as Bell-Kochen-Specker’s, which proves the *contextuality* of QM, and Bell’s, which proves the *contextuality at a distance*, or *nonlocality*, of QM. Indeed, nonobjectivity has some well known intriguing consequences.

(i) *Objectification problem in the quantum theory of measurement*. If QM is a universal theory, nonobjectivity extends to properties of macroscopic objects, against everyday evidence. This problem is illustrated by famous paradoxes, as Schrödinger’s cat, Wigner’s friend, etc.

(ii) *No intuitive model for QM can be provided (wave-particle duality)*. Every such model would indeed imply objectivity of all properties.

(iii) *Non-epistemic probability*. Quantum probability does not allow an ignorance interpretation, for the values of nonobjective properties cannot be assigned independently of a measurement context, that is, independently of observation. This feature of QM implies some interpretative problems (in particular, proper and improper mixtures have the same mathematical representation but different physical interpretations).

(iv) *Quantum truth and quantum logic*. The classical notion of truth as correspondence is unsuitable for the observational language of QM, for no extension made up of individual objects can be associated with a property that is nonobjective in a given state of  $\Omega$ . Hence, a non-classical notion of truth is required.

Notwithstanding the problematic consequences summarized above, contextuality and nonlocality are usually maintained to be distinguishing features of QM whenever one does not explicitly restrict to a statistical interpretation, independently of the foundational approach that is adopted (e.g., in the quantum logical, in the operational and in the algebraic approach, in Bohm’s theory, etc.). Moreover, nowadays quantum information theory considers contextual and nonlocal correlations as basic resources for quantum information processing and has inspired new foundational approaches, as Zeilinger’s [3], Clifton-Bub-

Halvorson’s [4], etc. It must be stressed that the acceptance of contextuality and nonlocality in these approaches stands not only on the “no-go” theorems but also on a series of experimental results that started with the famous Aspect’s experiments [5-7].

Philosophers of science know, however, that no set of experimental results may determine in a unique way a theory that explains them. Moreover, every “no-go” theorem follows from assumptions (some of which are often left implicit) that can be questioned. Several years ago a research was therefore started by the author, together with some collaborators, with the aim of inquiring whether it was possible to recover objectivity by embedding the mathematical apparatus of QM into a broader mathematical framework and reinterpreting it in such a way to turn around the “no-go” theorems. Of course, this new framework had to satisfy a basic requirement, that is, it had to explain the experimental results mentioned above and, more generally, the empirical success of QM. This research has been recently completed with the proposal of a new theory called *ESR (extended semantic realism) model* [8-17]. The main features of this model are resumed in Sect. 3, and some results obtained by discussing known problems of QM in the new framework are presented in Sect. 4. Sect. 2 is instead devoted to a preliminary clarification of the notion of nonobjectivity.

## 2 On the notion of nonobjectivity

To make the notion of nonobjectivity that will be used in this paper more precise, let us firstly recall that a physical system  $\Omega$  is usually associated in QM with a set  $\mathcal{S}$  of *states* and a set  $\mathcal{O}$  of *observables*. The set  $\mathcal{S}$  is partitioned into a subset  $\mathcal{P}$  of *pure* states and a subset  $\mathcal{M}$  of *mixtures*. Furthermore, a (*physical*) *property* is defined as a pair  $F = (A, \Sigma)$ , with  $A \in \mathcal{O}$  and  $\Sigma$  a Borel subset of the set  $\Xi$  of all possible values of  $A$  [1, 2]. The physical system  $\Omega$  can then be characterized by a triple  $(\mathcal{S}, \mathcal{F}, p)$  [1, 18]), where  $\mathcal{F}$  is the set of all properties of  $\Omega$  and  $p$  is a probability function

$$p : (S, F) \in \mathcal{S} \times \mathcal{F} \longrightarrow [0, 1].$$

Because of the characterization above, properties play a fundamental role in the foundations of QM. Given a property  $F = (A, \Sigma) \in \mathcal{F}$ , one says that  $F$  has truth value *true (false)* iff the value of  $A$  belongs (does not belong) to  $\Sigma$ . If one adopts the realistic interpretation of QM (Sect. 1), every property  $F$  is in principle measurable (but different properties may be not simultaneously measurable) on an individual object  $a$ , that is, an item of  $\Omega$ . The standard formulations of QM usually consider only *idealized* (efficiency 1) measurements. These measurements are dichotomic and their outcomes are labeled *yes* and *no*, the former corresponding to the value *true* of  $F$  and the latter to the value *false*.

The notion of objectivity can now be defined as follows.

*A property  $F$  of  $\Omega$  is objective for an individual object  $a$  iff its value (true/false) is not only assigned for every measurement context (value definiteness) but also independent of this context.*

It follows from the definition above that a property  $F$  is *nonobjective* whenever its value is not assigned for every measurement context or, if assigned, it depends on the context. Based on this definition one concludes that, if one does not want to reduce to a merely statistical interpretation, the realistic interpretations of QM imply that QM is a nonobjective theory, in the sense that, for every individual object  $a$  in a given state  $S$  of a physical system  $\Omega$ , there are both properties that can be considered objective and properties that must be considered nonobjective. To be precise, if an individual object  $\alpha$  is in the state  $S$ , then  $F$  can be considered objective for  $\alpha$  if  $p(S, F) \in \{0, 1\}$ , but it is necessarily nonobjective if  $p(S, F) \notin \{0, 1\}$  (note that this conclusion implies that  $F$  is objective for  $\alpha$  if and only if it is objective for every individual object in the state  $S$ ).

### 3 The ESR Model

The ESR model stems from the intuitive idea that the set of all properties which are objective for an individual object  $\alpha$  in a state  $S$  (to be determined by the model itself, as in QM) may be such that  $\alpha$  has nonzero probability of remaining undetected when a property  $F$  is measured on it. This “no-detection” probability may vary with  $F$  and  $S$  but does not depend on the device that is used to perform the measurement: hence, it may be different from 0 also in the case of exact (efficiency 1) measurements. The lack of efficiency of real measurements superimposes to it, usually hiding it.

#### 3.1 The fundamental assumption

To formalize the intuitive idea expounded above, the ESR model starts from the quantum description of a physical system  $\Omega$  in terms of states and observables, but adds a “no-registration outcome”  $a_0$  to the set  $\Xi$  of all possible values of any quantum observable  $A$ . The outcome  $a_0$  is considered as a possible result of an exact measurement of  $A$  and not only as the initial position of a pointer that is abandoned when the measurement is performed. Hence the introduction of  $a_0$  transforms the quantum observable  $A$  into a *generalized observable*  $A_0$ . This generalized observable is then associated with a family of properties of the form  $(A_0, \Sigma)$ , where  $\Sigma$  is a Borel subset of the set  $\Xi_0 = \Xi \cup \{a_0\}$  of all possible values of  $A_0$ . When  $a_0$  does not belong to  $\Sigma$ , the property  $F = (A_0, \Sigma)$  coincides with the quantum property  $(A, \Sigma)$ . Therefore the subset  $\{(A_0, \Sigma) \mid a_0 \notin \Sigma\}$  of all properties of this kind corresponds bijectively to the set of all properties of  $\Omega$  in QM and can be identified with it (hence it is denoted by  $\mathcal{F}$  in the following). Then, the intuitive idea expounded above can be formally expressed by the *fundamental equation* of the ESR model

$$p^t(S, F) = p^d(S, F)p(S, F).$$

In this equation  $S$  is a state and  $F = (A_0, \Sigma) \in \mathcal{F}$ . Then,  $p^t(S, F)$  is the overall probability that an idealized measurement of  $F$  performed on an

individual object  $\alpha$  in the state  $S$  yields outcome *yes*,  $p^d(S, F)$  is the probability that  $\alpha$  is detected in the measurement (*detection probability*), and  $p(S, F)$  is the probability that the measurement yields outcome *yes* when  $\alpha$  is detected (*conditional on detection probability*).

The *fundamental assumption* of the ESR model can now be stated as follows.

AX. *Let  $S \in \mathcal{P}$  and  $F \in \mathcal{F}$ . Then, the probability  $p(S, F)$  coincides with the probability supplied by QM, via Born's rule.*

It is important to note that assumption AX concerns pure states only (mixtures require indeed a separate treatment, see Sect. 3.3). Furthermore this assumption has two relevant consequences.

(i) *Conservative.* The ESR model embodies the mathematical formalism of QM.

(ii) *Innovative.* The ESR model deeply modifies the standard interpretation of the mathematical formalism of QM. According to QM, Born's rule supplies an absolute probability (physically interpreted as the large number limit of the ratio  $n/N$ , where  $n$  is the number of individual objects in the state  $S$  that display the property  $F \in \mathcal{F}$  when  $F$  is measured, and  $N$  is the number of individual objects in the state  $S$ ). According to the ESR model, if  $S$  is pure the same rule supplies a conditional probability (physically interpreted as the large number limit of the ratio  $n/N^d$ , where  $N^d \leq N$  is the number of all individual objects in the state  $S$  that are detected when  $F$  is measured).

## 3.2 The mathematical representation

For every  $S \in \mathcal{P}$  and  $F = (A_0, \Sigma) \in \mathcal{F}$ , the introduction of the three probabilities  $p^t(S, F)$ ,  $p^d(S, F)$  and  $p(S, F)$  in place of the standard quantum probability implies that the mathematical formalism of QM must be extended to take into account these probabilities. Such an extension leads to new representations of states, observables and properties.

*The detection probability  $p^d(S, F)$ .* No theory is available at present to predict  $p^d(S, F)$ . Hence  $p^d(S, F)$  is considered as a parameter in the ESR model, to be determined empirically. It is only required that  $p^d(S, F)$  satisfies a mathematical assumption (see below) that seems quite natural from an intuitive point of view.

*The conditional on detection probability  $p(S, F)$ .* Assumption AX implies that this probability can be obtained by using standard quantum rules. Hence, as far as  $p(S, F)$  is concerned, the physical system  $\Omega$  can be associated with a Hilbert space  $\mathcal{H}$ . Moreover, a pure state  $S$  can be represented by a one-dimensional orthogonal projection operator  $\rho_S$  on  $\mathcal{H}$ , the generalized observable  $A_0$  can be represented by the same self-adjoint operator  $\hat{A}$  that represents the observable  $A$  of QM from which  $A_0$  is obtained, and the property  $F$  can be represented by an orthogonal projection operator  $P^{\hat{A}}(\Sigma)$  on  $\mathcal{H}$ . Furthermore, the standard quantum equation holds

$$p(S, F) = \text{Tr}[\rho_S P^{\hat{A}}(\Sigma)].$$

The overall probability  $p^t(S, F)$ . Bearing in mind the fundamental equation of the ESR model and the mathematical representation of  $p_S(F)$ , one obtains

$$p^t(S, F) = \text{Tr}[p^d(S, F)\rho_S P^{\hat{A}}(\Sigma)].$$

Hence

$$p^t(S, F) = \text{Tr}[\rho_S T_{S, A_0}(\Sigma)],$$

where  $T_{S, A_0}(\Sigma) = p^d(S, F)P^{\hat{A}}(\Sigma)$  is a positive operator bounded by 0 and 1 (*effect*). One then assumes that a mapping  $p_{S, A_0}^d(\lambda)$  of the set  $\Xi$  of all possible values of  $A$  into  $[0, 1]$  exists such that

$$T_{S, A_0}(\Sigma) = \int_{\Sigma} p_{S, A_0}^d(\lambda) P^{\hat{A}}(d\lambda)$$

(the existence of  $p_{S, A_0}^d(\lambda)$  constitutes the only mathematical assumption on  $p^d(S, F)$  in the ESR model).

The above equations imply that, as far as  $p^t(S, F)$  is concerned, the pure state  $S$  can still be represented by  $\rho_S$  and the property  $(A_0, \Sigma)$  is represented by a family  $\{T_{S, A_0}(\Sigma)\}_{S \in \mathcal{P}}$  of effects. Moreover, the generalized observable  $A_0$  is represented by the family of *commutative operator valued measures*

$$\mathcal{T}_{A_0} = \{T_{S, A_0} : \Sigma \in B(\Xi) \longrightarrow T_{S, A_0}(\Sigma) \in \mathfrak{B}(\mathcal{H})\}_{S \in \mathcal{P}}$$

where  $B(\Xi)$  is the set of all Borel sets on  $\Xi$  and  $\mathfrak{B}(\mathcal{H})$  the set of all bounded positive operators on  $\mathcal{H}$ .

Putting together the representations of properties to be used in order to evaluate the conditional on detection probability  $p_S(F)$  and the overall probability  $p_S^t(F)$  in the case of pure states, one obtains that a complete mathematical representation of a property  $F = (A_0, \Sigma) \in \mathcal{F}$  is provided in the ESR model by the pair

$$(P^{\hat{A}}(\Sigma), \{T_{S, A_0}(\Sigma)\}_{S \in \mathcal{P}}).$$

Analogously, a complete representation of the generalized observable  $A_0$  is provided by the pair

$$(\hat{A}, \mathcal{T}_{A_0}) = (\hat{A}, \{T_{S, A_0} : \Sigma \in B(\Xi) \longrightarrow T_{S, A_0}(\Sigma) \in \mathfrak{B}(\mathcal{H})\}_{S \in \mathcal{P}}).$$

The following remarks are then important.

(i) In the representation of  $F$  the first element of the pair coincides with the standard representation of  $F$  in QM. In the representation of  $A_0$  the first element of the pair coincides with the standard representation of the quantum observable  $A$  from which  $A_0$  is obtained.

(ii) In both representations the second element is a family, parametrized by the set of pure states. Hence, as far as  $p^t(S, F)$  is concerned, the representation of a property, or of an observable, is not given once for all, because it depends on the state of the individual object on which the property, or the observable, is measured.

### 3.3 Proper and improper mixtures

The results expounded in Sect.3.2 show that pure states can be represented in the ESR model by the same density operators that represent them in QM. One can then wonder whether similar results hold in the case of mixtures.

According to many authors [1, 18, 19] there are in QM *proper* and *improper* mixtures, which are mathematically represented in the same way (density operators) but have different operational definitions, which imply different interpretations of the coefficients that occur in their decompositions in terms of pure states (epistemic versus nonepistemic probabilities).

In the ESR model these two kinds of mixtures have different mathematical representations, corresponding to their different operational definitions [10, 12, 13, 17], as follows.

(i) *Improper mixtures*. These mixtures can be represented by the same density operators that represent them in QM. Assumption AX can be extended to improper mixtures by substituting the subset  $\mathcal{P}$  of all pure states with the subset  $\mathcal{P} \cup \mathcal{N}$ , where  $\mathcal{N}$  is the subset of all improper mixtures. The representations of properties and observables can then be extended to improper mixtures by introducing the same substitution. Hence improper mixtures are considered as *generalized pure states* in the ESR model.

(ii) *Proper mixtures*. Each proper mixture has a rather complicated representation as a family of pairs parametrized by the set  $\mathcal{F}$  of properties. Each pair in the family consists of a density operator and a detection probability. The explicit form of these mathematical entities is given in [10, 12, 13] and will not be reported here for the sake of brevity.

### 3.4 The generalized Lüders postulate

In QM the Lüders postulate selects a subset of (exact) *ideal first kind measurements* that change a state according to a prefixed rule [2]. This postulate is generalized in the ESR model as follows.

Consider an exact dichotomic measurement  $\mathfrak{M}$  of a property  $F = (A_0, \Sigma) \in \mathcal{F}$  on an individual object  $\alpha$  in the state  $S$ , with  $S$  a pure state or an improper mixture. Then  $\mathfrak{M}$  is an *idealized* measurement of  $F$  if the state  $S_F$  after the measurement is represented by the density operator

$$\rho_{S_F} = \frac{T_{S, A_0}(\Sigma) \rho_S T_{S, A_0}^\dagger(\Sigma)}{\text{Tr}[T_{S, A_0}(\Sigma) \rho_S T_{S, A_0}^\dagger(\Sigma)]}$$

whenever the *yes* outcome is obtained,.

By analogy with QM, the rule expressed by the equation above is called *the generalized Lüders postulate*. It must be stressed that it does not apply directly to proper mixtures (which are not represented as in QM, see Sect. 3.3). However, the representation of the final state in the case of proper mixtures can be deduced from the equation above. Its mathematical form is rather complicated [10, 12, 13] and will not be reported here for the sake of simplicity.

### 3.5 Time evolution

The ESR model has been recently completed by studying time evolution [17], based on the idea that the generalized Lüders postulate supplies an example of the change of state of an individual object interacting with another object (the measuring apparatus). Indeed this example provides some suggestions for the dynamics of the composite system of the two objects. In particular, a crucial difference from time evolution in QM occurs because the generalized Lüders postulate introduces a change of state also in the case of individual objects that are not detected by the measurement.

The following conclusions are attained in the case of pure states or improper mixtures (the details of the treatment will not be reported here for the sake of brevity).

- (i) One can assume that closed systems undergo linear Hamiltonian evolution, as in QM.
- (ii) The evolution of open systems may be linear or not, depending on their interaction with the environment, as in QM.
- (iii) The evolution induced by a measurement on an individual object is necessarily nonlinear.

The results above show that time evolution in the ESR model matches time evolution in QM, but for the distinguishing feature in item (iii). One can then prove that time evolution in the case of proper mixtures can be deduced from time evolution in the case of pure and generalized pure states if an obvious assumption is introduced.

### 3.6 H.V. models and objectivity

It remains to discuss the crucial issue of nonobjectivity. Indeed, the main aim of the ESR model is supplying an objective theory, embodying from one side the basic mathematical formalism of QM and avoiding, on the other side, the problems following from nonobjectivity (Sect. 1).

The proof of the objectivity of the ESR model is obtained by showing that this model admits noncontextual (hence local) *hidden variables (h.v.) models* (at variance with earlier formulations [8-16], the latest version of the ESR model [17] does not introduce h.v. from the beginning). To this end a set  $\mathcal{F}_\mu$  of *microscopic properties* of the physical system  $\Omega$  is introduced which is in one-to-one correspondence with the set  $\mathcal{F}$  of (macroscopic) properties. For every individual object  $\alpha$ , the set  $\mathcal{F}_\mu$  is then partitioned into two subsets, the subset  $s$  of all the microscopic properties that are *possessed* by  $\alpha$  and the subset  $\mathcal{F}_\mu \setminus s$  of all the microscopic properties that are *not possessed* by  $\alpha$ . The subset  $s$  is called *the microscopic state* of  $\alpha$ . Then, new overall probability, detection probability and conditional on detection probability are introduced referring to the microscopic state  $s$  of  $\alpha$  rather than to its (macroscopic) state  $S$ . By introducing the further probability  $p(S | s)$  that an individual object  $\alpha$  in the state  $S$  is in the microscopic state  $s$ , one can deduce the fundamental equation of the ESR model, thus obtaining the desired noncontextual h.v. model.



Because of the above result and of the one-to-one correspondence between  $\mathcal{F}_\mu$  and  $\mathcal{F}$ , one concludes that all properties in  $\mathcal{F}$  can be considered objective in the sense specified in Sect. 1. Hence the ESR model is an objective theory. It follows in particular that quantum probabilities can be considered epistemic, so that no objectification problem occurs. Of course, this result finds its roots in the reinterpretation of quantum probabilities as conditional on detection rather than absolute (Sect. 3.1), which allows to turn around the “no-go” theorems of QM (Sect. 4.1).

### 3.7 Empirical consequences

As we have anticipated in Sect. 1, the empirical success of QM imposes a fundamental constraint on every attempt at modifying QM to avoid the problems following from nonobjectivity. The predictions of QM that have been experimentally verified must in fact be reproduced by the new theory within the limits of the experimental errors. On the other side, the new theory should also provide some testable predictions that make it empirically different from QM, allowing one to check which theory is correct. The ESR model satisfies both these conditions. Indeed, the predictions of the ESR model in experiments on overall probabilities are formally different from the predictions of QM, but, if the state  $S$  of the individual objects that are considered is a pure state or an improper mixture, they may be close to the quantum predictions whenever the detection probabilities are close to 1. Moreover the predictions of the ESR model in experiments on conditional on detection probabilities (as Aspect’s experiments, in which non-detected individual objects are not taken into account [5, 6]) are identical to the predictions of QM. The predictions of the ESR model in experiments on overall probabilities in which the state  $S$  of the individual objects that are considered is a proper mixture may be instead very different from the predictions of QM and single out a class of experiments that can distinguish the two theories.

## 4 Applications

The ESR model has been used to deal with some well known problematical issues in QM. The obtained results can be resumed as follows.

### 4.1 The “no-go” theorems

Because of Assumption AX, the “no-go” theorems of QM do not hold in the ESR model [9, 11, 16]. This relevant result can be intuitively explained as follows.

By considering only the Bell-KS and Bell theorems for the sake of brevity, one sees that all proofs that do not resort to inequalities proceed *ab absurdo*. They consider some different quantum laws linking together physical properties of an individual object  $\alpha$  and show that a contradiction occurs if all properties of  $\alpha$  are supposed to be objective. The laws that are chosen, however, cannot be

checked simultaneously. Indeed each of them contains some observables that are incompatible with some of the observables that occur in the other laws. Suppose that a measurement is performed on  $\alpha$  to check one of the laws, and that  $\alpha$  is detected. Then, the law will be confirmed. But one cannot simultaneously check some of the remaining laws, and cannot exclude that the objective properties of  $\alpha$  be such that  $\alpha$  would not be detected if such a check were done. Thus, the assumption that all laws must simultaneously hold for  $\alpha$  is arbitrary in the framework of the ESR model, and no “conspiracy of nature” is required to reach this conclusion. Hence the aforesaid proofs of contextuality and nonlocality, that imply nonobjectivity, rest on a questionable assumption from the point of view of the ESR model.

A similar line of argument holds when considering the proofs of the Bell theorem that resort to inequalities. Indeed Bell’s inequalities do not hold in the ESR model at a macroscopic level, notwithstanding nonobjectivity (they hold instead in the h.v. models discussed in Sect. 3.6, at a purely theoretical microscopic level). When considering, for instance, the original Bell’s inequality, one obtains that it must be replaced by the equation

$$|E(A_0(a), B_0(b)) - E(A_0(a), B_0(c))| \leq 1 + E(A_0(b), B_0(c))$$

where  $E(A_0(a), B_0(b))$  denotes the expectation value of the products of the *trichotomic* observables  $A_0(a)$  and  $B_0(b)$ , depending on the parameters  $a$  and  $b$ , respectively. The symbols  $E(A_0(a), B_0(c))$  and  $E(A_0(b), B_0(c))$  have similar meanings.

Analogously, the Clauser-Horne-Shimony-Holt inequality must be replaced by the equation

$$|E(A_0(a), B_0(b)) - E(A_0(a), B_0(c))| + |E(A_0(d), B_0(b)) - E(A_0(d), B_0(c))| \leq 2.$$

These modified Bell’s inequalities, do not necessarily contrast with quantum inequalities. Hence also the proofs of nonlocality resorting to inequalities are invalid in the ESR model.

Rather than objectivity, the ESR model questions the unrestricted validity of quantum laws. Assumption AX implies indeed that a quantum law holds for an individual objects  $\alpha$  if  $\alpha$  is detected when the law is checked on it, while it does not necessarily hold if  $\alpha$  remains undetected because of its objective properties (that are not uniquely determined by the state  $S$  of  $\alpha$  in the ESR model).

## 4.2 The GHZ experiment

The general h.v. models for the ESR model can be used to produce h.v. models for specific physical situations and experiments.

In particular, it has been recently proved that the finite “toy models” contrived by Szabó and Fine in 2002 to provide a local explanation of the Greenberger-Horne-Zeilinger (GHZ) experiment can be obtained as special cases of the foregoing general h.v. models [14].

### 4.3 Quantum logic and quantum truth

It has also been recently shown that quantum logic can be embedded into a suitable extended classical logic, the embedding preserving the logical order but not the algebraic structure [15].

The above result must be considered as purely formal if one accepts the standard interpretation of QM. It acquires instead a physical interpretation in the ESR model because of objectivity of properties in this model. Objectivity indeed allows one to consider the set of individual objects formally associated with every  $F \in \mathcal{F}$  as the set of all objects that *possess* the property  $F$ . It follows that no notion of quantum truth, different from classical truth and incompatible with it is needed in the ESR model. Rather, quantum logic can be seen as a mathematical structure formalizing the metalinguistic notion of *verifiability* according to QM.

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